**Barron’s Let’s Review Regents – Algebra II**

# Chapter 8: Sequences

## 8.1 Sequences and Series

**Key Ideas**

A sequence is a list of numbers that follows some pattern. An example of a sequence is The individual terms of the sequence are called , where the small subscript is the term’s position on the list. The terms of the sequence can be defined with two types of formulas. An explicit formula describes how to calculate the term by knowing the term’s position on the list. A recursive formula describes how to calculate the term by knowing the previous terms on the list. The explicit formula is generally preferred but more difficult to find.

**Explicit Formula for a Sequence**

An explicit formula is a type of function into which only positive integers can be input. Sometimes the explicit formula notation resembles function notation like . An alternative notation is with a subscript, like . By substituting the values 1, 2, 3, and 4 for , the first four terms of the sequence can be determined.

The sequence is

**Math Facts**

An *infinite sequence* is one that has no end. The ellipses (…) at the end indicates that the sequence is infinite. A *finite sequence* does have a final term. The sequence 5, 7, 9, 11 is a finite sequence.

**Example 1**

What are the first three terms of the sequence defined by equation ?

*Solution*: Substitute 1,2, and 3 for in the formula to get

**Recursive Formula for a Sequence**

A recursive formula has two parts. First there is the base case, where the first value (or the first few values) of the sequence are given. Second, there is the *recursive definition*, where a rule for calculating ore terms is described in terms of the previous terms.

For example:

To get , there is no calculation needed since is given as 5 in the base case. For , substitute 2 for into the recursive part of the definition.

Notice that the value of needs to be known to do this step.

For

The first three terms are 5, 7 and 9.

**Example 2**

What is the value of for the sequence defined by

Solution: is given.

Substitute 2 for into the recursive part of the definition.

Substitute 3 for into the recursive part of the definition.

**Example 3**

What is the term of the sequence defined recursively as the following?

**Solution**:

Each term becomes . So .

**Arithmetic and Geometric Sequences**

A sequence like 5, 7, 9, 11, … is known as an arithmetic sequence since each term is equal to 2 more than the previous term. The number 2, in this case, is known as the *common difference*, usually denoted by the variable .

A sequence like 5, 10, 20, 40, … is known as a *geometric sequence* since each term is equal to 2 times the previous term. The number 2, in this case, is known as the common ratio, usually denoted by the variable .

The two formulas for the th term of an arithmetic or of a geometric sequence are provided on the reference sheet given to you in the Regents booklet.

|  |  |
| --- | --- |
| **Arithmetic Sequence** | **Geometric Sequence** |
|  |  |

**Example 4**

Find the explicit formula for the term term of sequence 7, 11, 15, 19, … .

*Solution*: Since each term is equal to the previous term plus 4, this is an arithmetic sequence. . The formula is .

**Example 5**

Find the explicit formula for the term of sequence 7, 28, 112, 448, … .

*Solution*: Since each term is equal to the previous term times 4, this is a geometric sequence: . The formula is .

**Example 6**

What is the 50th term of the sequence 7, 11, 15, 19, …?

Solution: The explicit formula is . Substitute 50 for to get the answer.

Example 7

The sequence will eventually reach the number 251. If , what is the value of ?

Solution: The formula is . If   
, the equation becomes

Now divide both sides of the equation by 8.

Then add 1 to both sides of the equation.

**Finite Arithmetic and Geometric Series**

A finite series is like a finite sequence except all the terms are added together. An example of a finite arithmetic series is . An example of a finite geometric series is   
.

The formula for calculating the sum of a finite geometric series is given in the reference sheet of the Regents booklet. The formula is show here.

To use this formula to calculate , substitute 6 for (since there are 6 terms), 5 for , and 2 for .

The formula for calculating the sum of a finite arithmetic series is not given in the Regents booklet. That formula is shown here.

**Example 8**

What is the sum of the first 100 positive integers in the following series?

Solution: Substitute 100 for , 1 for , and 100 for .

### Check Your Understanding of Section 8.1

1. *Multiple-Choice*
2. What are the first four terms of the sequence defined by for ?  
   **(1) 5, 8, 11, 14**
3. The sequence 3, 12, 48, 192, … can be defined by which of the following?  
   **(2)**
4. What is the twentieth term of the sequence 4, 11, 18, 25, …?  
   **(2) 137**
5. What position is the number 247 in the sequence 4, 13, 22, 31, …?  
   **(3) 28**
6. What is the 10th term of the sequence 4, 20, 100, 500, …?  
   **(1) 7,812,500**
7. What is the sum of the series   
   ?  
   **(4) 6,440**
8. What is the sum of the series   
   ?  
   **(4)**
9. A series is defined as   
    for . What is the value of ?  
   **(3) 44**
10. What is the sum of the first 20 terms of   
    ?  
    **(3)**
11. is closest to the following?  
    **(3) 0.9999995**
12. *Show how you arrived at your answers*.
13. For 20 years, Journey puts $1,000 into the bank every January 1. The bank pays 3% (5%?) interest every December 31. If she starts on January 1, 2000, the amount of money she will have on January 2, 2020 can be shown by the sequence:
14. Which is greater: the 20th term of the sequence 10,000; 20,000; 30,000; 40,000; … or the 20th term of the sequence 1, 2, 4, 8, …?

Sequence 1:  
The 20th term of the sequence 1, 2, 4, 8, … is greater.

1. The Fibonacci sequence is defined by   
   . What is ?

|  |  |
| --- | --- |
|  | 1 |
|  | 1 |
|  | 2 |
|  | 3 |
|  | 5 |
|  | 8 |
|  | 13 |
|  | 21 |
|  | 34 |
|  | 55 |

1. If and , what is ?

|  |  |
| --- | --- |
|  | 3 |
|  | 14 |
|  | 129 |
|  | **2,068** |

**My answer does not agree with the book’s answer of 2,356.**

1. 2,657,205 is what term in the sequence 5, 15, 45, 135, …?  
   Check: